

塾技 37 因数分解の応用 (3)

問題 1 (難易度 B)

$x^2 + y^2 - 2xy - 3x + 3y$ を因数分解せよ。

(近畿大附高)

問題 2 (難易度 B)

$a^2 + b^2 - 2a - 2ab + 2b + 1$ を因数分解せよ。

(関西学院高)

問題 3 (難易度 C)

$a^3 - 2a^2b + ab^2 + a^2c - abc$ を因数分解せよ。

(明治学院高)

問題 4 (難易度 C)

$a^2 + b^2 - 3c^2 + 2(ab - bc - ca)$ を因数分解せよ。

(早稲田大本庄高)

解 1

$$\begin{aligned}
 & \overset{\textcircled{1}}{x^2} + \overset{\textcircled{2}}{y^2} - \overset{\textcircled{3}}{2xy} - \overset{\textcircled{4}}{3x} + \overset{\textcircled{5}}{3y} \\
 &= \overset{\textcircled{1,3,2}}{x^2 - 2xy + y^2} - \overset{\textcircled{4,5}}{3x + 3y} \\
 &= (x-y)^2 - 3(x-y) \\
 &= A^2 - 3A \\
 &= A(A-3) \\
 &= (x-y)(x-y-3) \quad \leftarrow \text{答}
 \end{aligned}$$

$\left. \begin{array}{l} \text{5項を3項} \cdot \text{2項} \\ \text{に分ける} \\ \text{部分的に因数分解} \\ \text{して共通因数を作る} \\ \text{ } \end{array} \right\} x-y=A \text{ とおく}$

解 2

$$\begin{aligned}
 & \overset{\textcircled{1}}{a^2} + \overset{\textcircled{2}}{b^2} - \overset{\textcircled{3}}{2a} - \overset{\textcircled{4}}{2ab} + \overset{\textcircled{5}}{2b} + \overset{\textcircled{6}}{1} \\
 &= \overset{\textcircled{1,4,2}}{a^2 - 2ab + b^2} - \overset{\textcircled{3,5}}{2a + 2b} + \overset{\textcircled{6}}{1} \\
 &= (a-b)^2 - 2(a-b) + 1 \\
 &= X^2 - 2X + 1 \\
 &= (X-1)^2 \\
 &= (a-b-1)^2 \quad \leftarrow \text{答}
 \end{aligned}$$

$\left. \begin{array}{l} \text{6項を3項} \cdot \text{2項} \\ \text{} \cdot \text{1項に分ける} \\ \text{部分的に} \\ \text{因数分解} \\ \text{ } \end{array} \right\} a-b=X \text{ とおく}$

解 3 「塾技 35 手順①」より、まず

共通因数 a をくくり出す。

$$\begin{aligned}
 & a^3 - 2a^2b + ab^2 + a^2c - abc \\
 &= a(a^2 - 2ab + b^2 + ac - bc) \\
 &= a(\overset{\textcircled{1}}{a^2} - \overset{\textcircled{2}}{2ab} + \overset{\textcircled{3}}{b^2} + \overset{\textcircled{4}}{ac} - \overset{\textcircled{5}}{bc}) \\
 &= a(\overset{\textcircled{1,2,3}}{a^2 - 2ab + b^2} + \overset{\textcircled{4,5}}{ac - bc}) \\
 &= a\{(a-b)^2 + c(a-b)\} \\
 &= a(X^2 + cX) \\
 &= aX(X+c) \\
 &= a(a-b)(a-b+c) \quad \leftarrow \text{答}
 \end{aligned}$$

$\left. \begin{array}{l} \text{5項を3項} \cdot \text{2項} \\ \text{に分ける} \\ \text{部分的に因数分解} \\ \text{して共通因数を作る} \\ \text{ } \end{array} \right\} a-b=X \text{ とおく}$
 $\left. \begin{array}{l} \text{ } \\ \text{ } \\ \text{ } \\ \text{ } \\ \text{ } \end{array} \right\} X \text{ をくくり出す}$

解 4

$$\begin{aligned}
 & \overset{\textcircled{1}}{a^2} + \overset{\textcircled{2}}{b^2} - \overset{\textcircled{3}}{3c^2} + \overset{\textcircled{4}}{2ab} - \overset{\textcircled{5}}{2bc} - \overset{\textcircled{6}}{2ca} \\
 &= \overset{\textcircled{1,4,2}}{a^2 + 2ab + b^2} - \overset{\textcircled{6,5}}{2ca - 2bc} - \overset{\textcircled{3}}{3c^2} \\
 &= (a+b)^2 - 2c(a+b) - 3c^2 \\
 &= X^2 - 2cX - 3c^2 \\
 &= (X-3c)(X+c) \\
 &= (a+b-3c)(a+b+c) \quad \leftarrow \text{答}
 \end{aligned}$$

$\left. \begin{array}{l} \text{6項を3項} \cdot \text{2項} \cdot \text{1項} \\ \text{に分ける} \\ \text{部分的に} \\ \text{因数分解} \\ \text{ } \\ \text{ } \\ \text{ } \\ \text{ } \end{array} \right\} a+b=X \text{ とおく}$